14.5 Differentiation of a vector fxn of a single variable

★ $R(\tau)$ 與 $R'(\tau)$ 垂直

(1) $\vec{R}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$ in Cartesian coordinate

$\nearrow$ 1D $R \in !$

$= r(\tau)\,\hat{e}_r + \theta(t)\,\hat{e}_\theta + z(t)\,\hat{k}$ in Cylindrical :

$= \rho(t)\,\hat{e}_\rho + \phi(t)\,\hat{e}_\phi + \theta(\tau)\,\hat{e}_\theta$ in Spherical :

2. (1) $\dfrac{d\vec{R}(\tau)}{d\tau} = \vec{R}'(\tau) \equiv \lim\limits_{\Delta\tau \to 0} \dfrac{\vec{R}(\tau+\Delta\tau) - \vec{R}(\tau)}{\Delta\tau}$;

tangent vector

(2) $|\vec{R}'(\tau)| = S'(\tau)$ $S(\tau)$: length of arc in ch15 #

< Ex $r(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}$

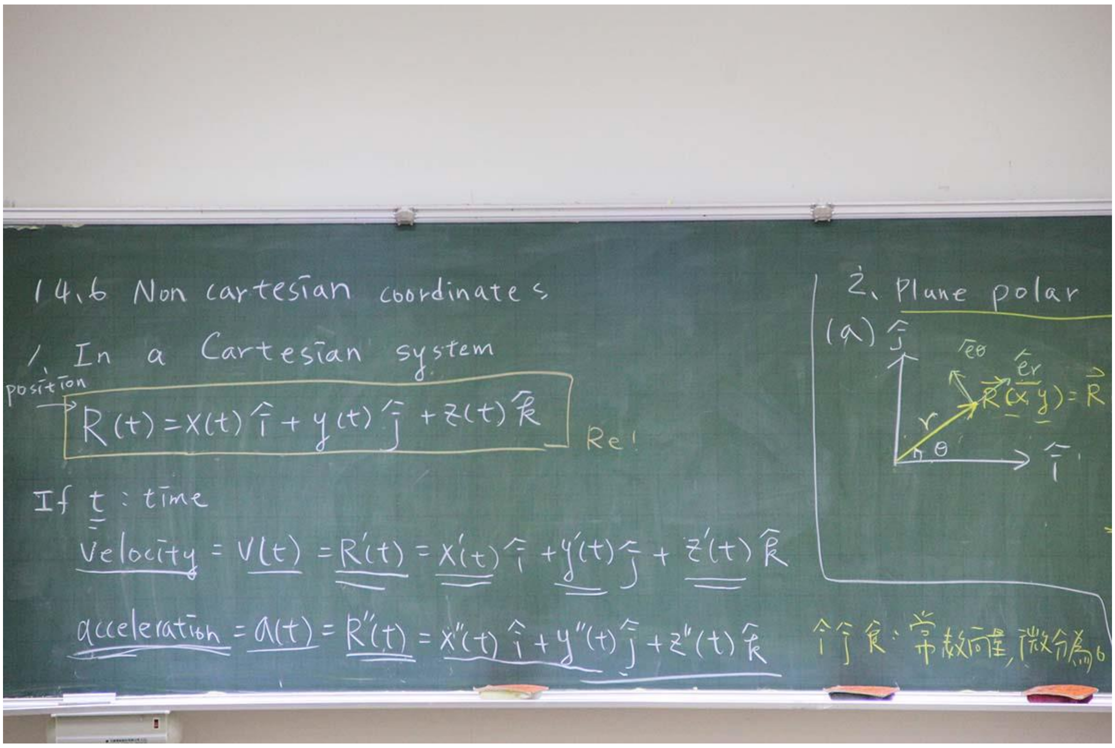
find $r'(t)$; $\hat{e}_t = ?$ $\dfrac{\vec{R}'(\tau)}{|\vec{R}'(\tau)|} = \hat{e}_t$

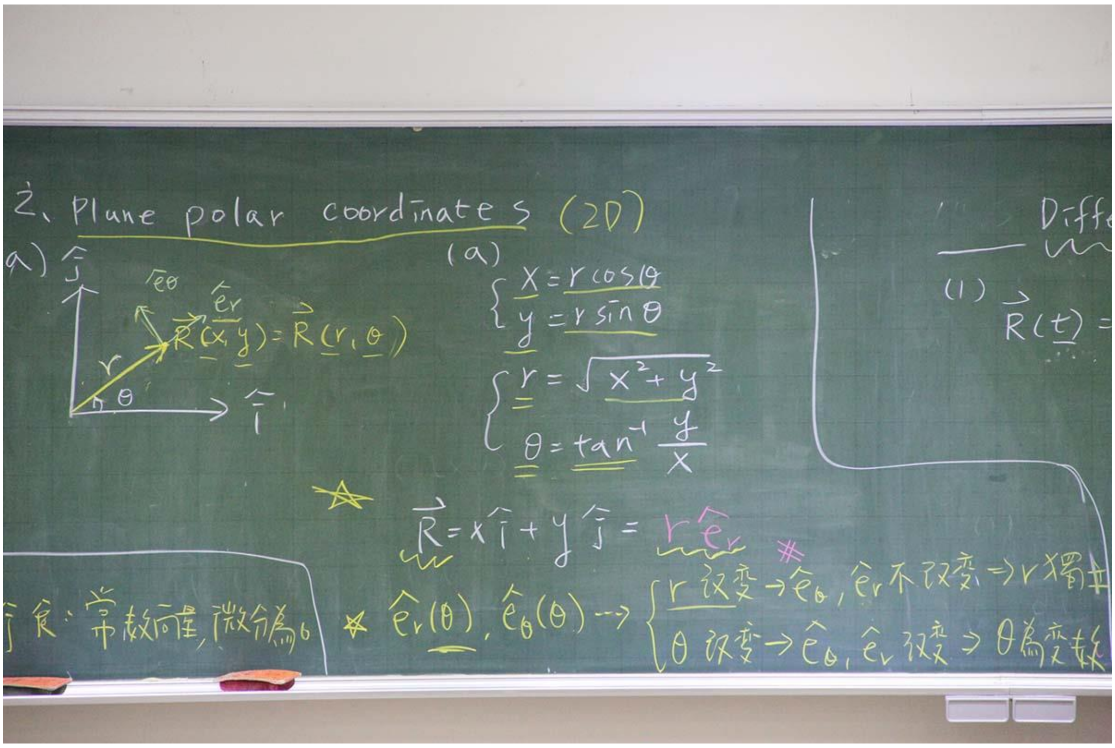
Sol

$r'(t) = -\sin t\,\hat{i} + \cos t\,\hat{j} + \hat{k}$

$S'(t) = |r'(t)| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$

$\hat{e}_t = \dfrac{r'(t)}{S'(t)} = \dfrac{-\sin t}{\sqrt{2}}\,\hat{i} + \dfrac{\cos t}{\sqrt{2}}\,\hat{j} + \dfrac{1}{\sqrt{2}}\,\hat{k}$


14.6 Non cartesian coordinates
1. In a Cartesian system
position
$R(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$
Re!
If t : time
Velocity = $V(t) = R'(t) = x'(t)\hat{i} + y'(t)\hat{j} + z'(t)\hat{k}$
acceleration = $a(t) = R''(t) = x''(t)\hat{i} + y''(t)\hat{j} + z''(t)\hat{k}$
2. Plane polar
(a) $\hat{j}$
$\hat{e}_\theta$ $\hat{e}_r$ $\vec{R}(x,y) = \vec{R}$
r θ $\hat{i}$
$\hat{i}\hat{j}\hat{k}$: 常數向量, 微分為0


2. Plane polar coordinates (2D)
(a) $\hat{j}$
$\hat{e}_\theta$ $\hat{e}_r$ $\vec{R}(x,y) = \vec{R}(r,\theta)$
r θ $\hat{i}$
(a)
$x = r\cos\theta$, $y = r\sin\theta$
$r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}\frac{y}{x}$
$\vec{R} = x\hat{i} + y\hat{j} = r\hat{e}_r$
$\hat{j}\hat{k}$: 常數向量, 微分為0
$\hat{e}_r(\theta), \hat{e}_\theta(\theta)$ --> r 改变 → $\hat{e}_\theta, \hat{e}_r$ 不改变 ⇒ r 獨立
θ 改变 → $\hat{e}_\theta, \hat{e}_r$ 改变 ⇒ θ 為变数
Diffe
(1) $\vec{R}(t)$ =

Differentiation of a vector fxn of a single variable

★ R(t)與R'(t)垂直, when |R(t)| is const ★

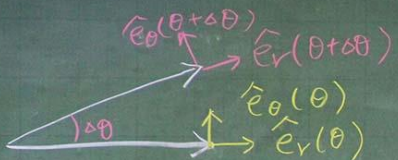
(1) $\vec{R}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$ — Cartesian coordinate

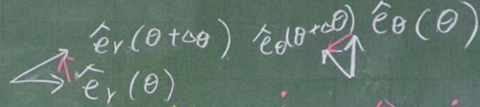
(b) $R(t) = r(t)\,\hat{e}_r(\theta(t))$

$$v(t) = \dot{r}\,\hat{e}_r + r\,\dot{\hat{e}}_r\,\dot{\theta}$$

$$= \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

＊計算





$\dot{\hat{e}}_r = \hat{e}_\theta\,\dot{\theta}$ $\dot{\hat{e}}_\theta = -\hat{e}_r\,\dot{\theta}$ Re!

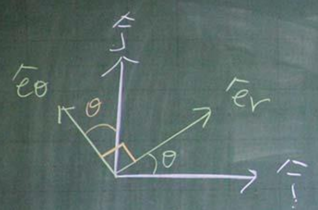
$$a(t) = v'(t) = \ddot{r}\,\hat{e}_r + \dot{r}\,\hat{e}_\theta\,\dot{\theta} + \dot{r}\,\dot{\theta}\,\hat{e}_\theta$$

$$+ r\ddot{\theta}\,\hat{e}_\theta - r\dot{\theta}\dot{\theta}\,\hat{e}_r$$

$$= (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\,\hat{e}_\theta$$

$\boxed{HW}$

(c) Transform method：轉換成 ($\hat{i}, \hat{j}, \hat{k}$) 後微分



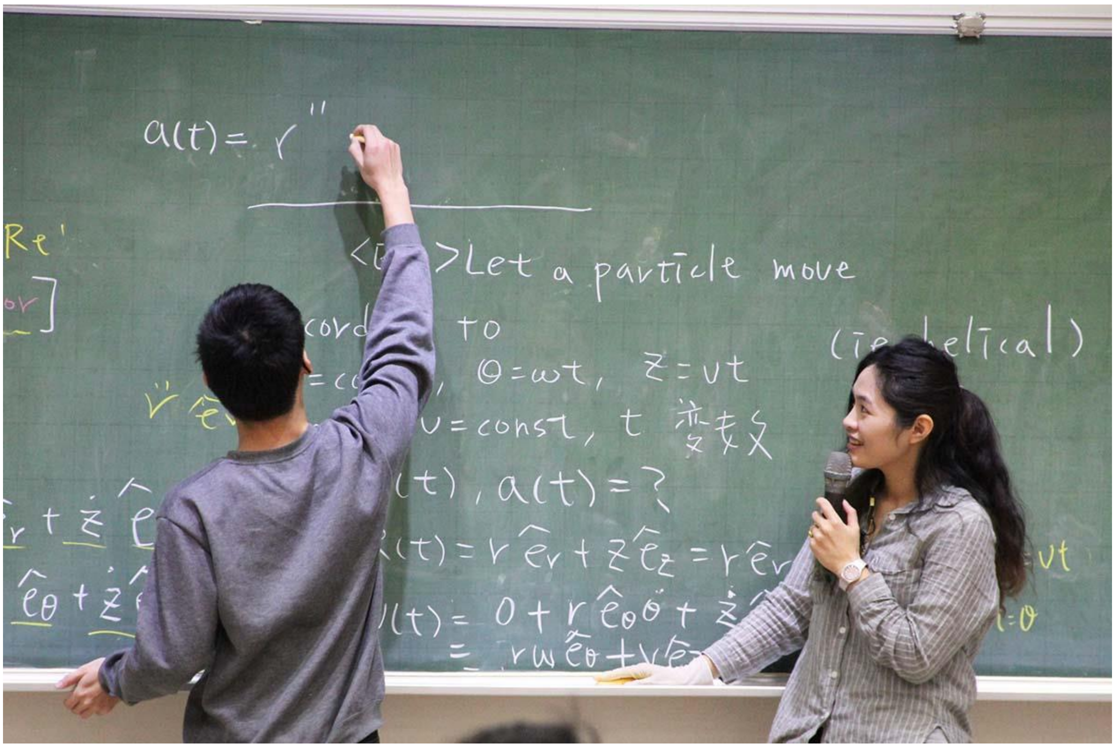
$$\begin{cases} \hat{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j} = \hat{e}_r(\theta) \\ \hat{e}_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j} = \hat{e}_\theta(\theta) \end{cases}$$

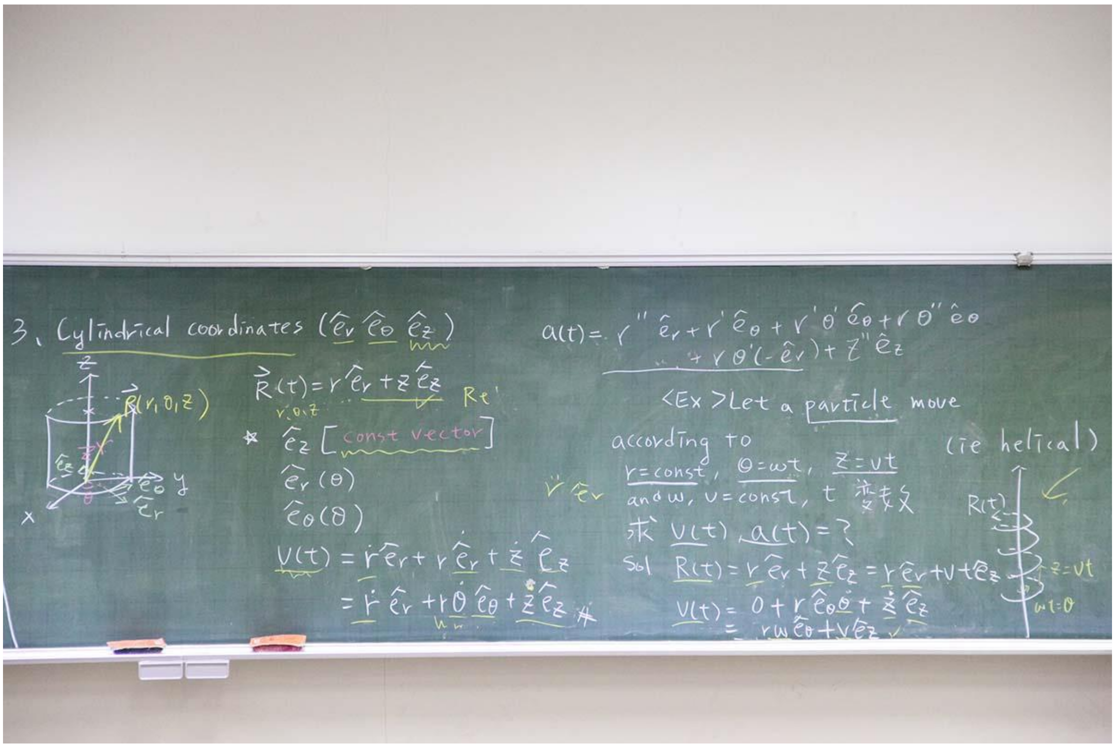
Review [$\hat{i}$ $\hat{j}$ ON basis]

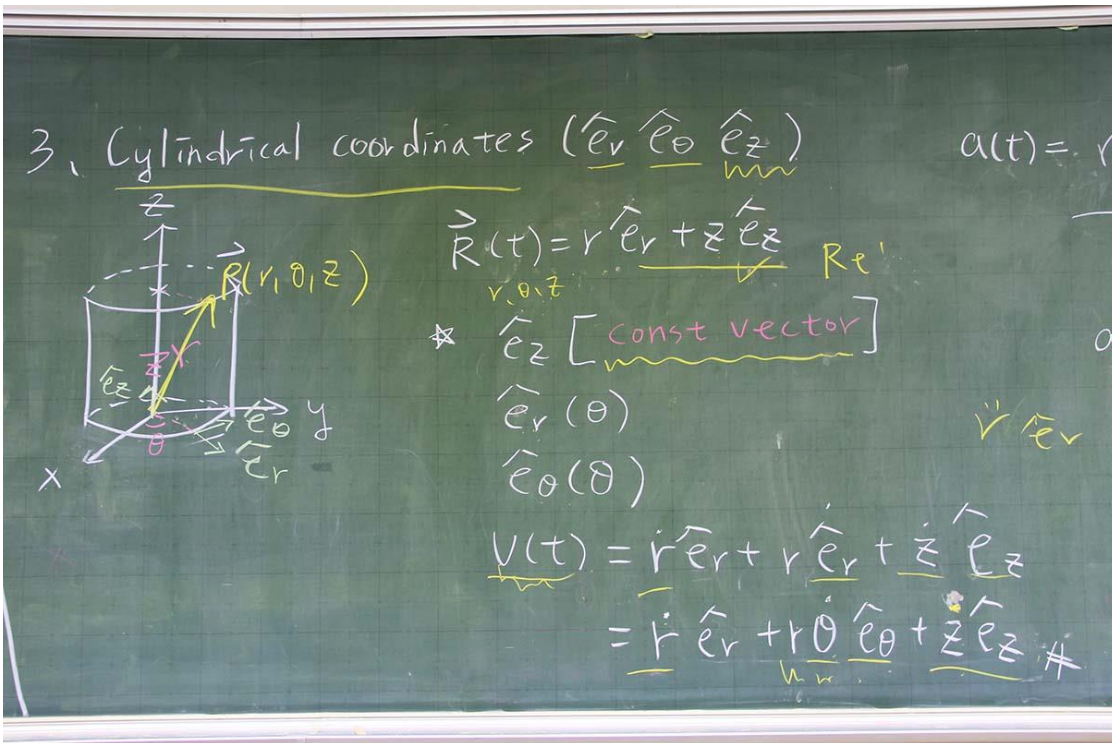
$$\hat{e}_r = [\hat{e}_r \cdot \hat{i}]\,\hat{i} + [\hat{e}_r \cdot \hat{j}]\,\hat{j} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\frac{d\hat{e}_r}{d\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j} = \hat{e}_\theta$$

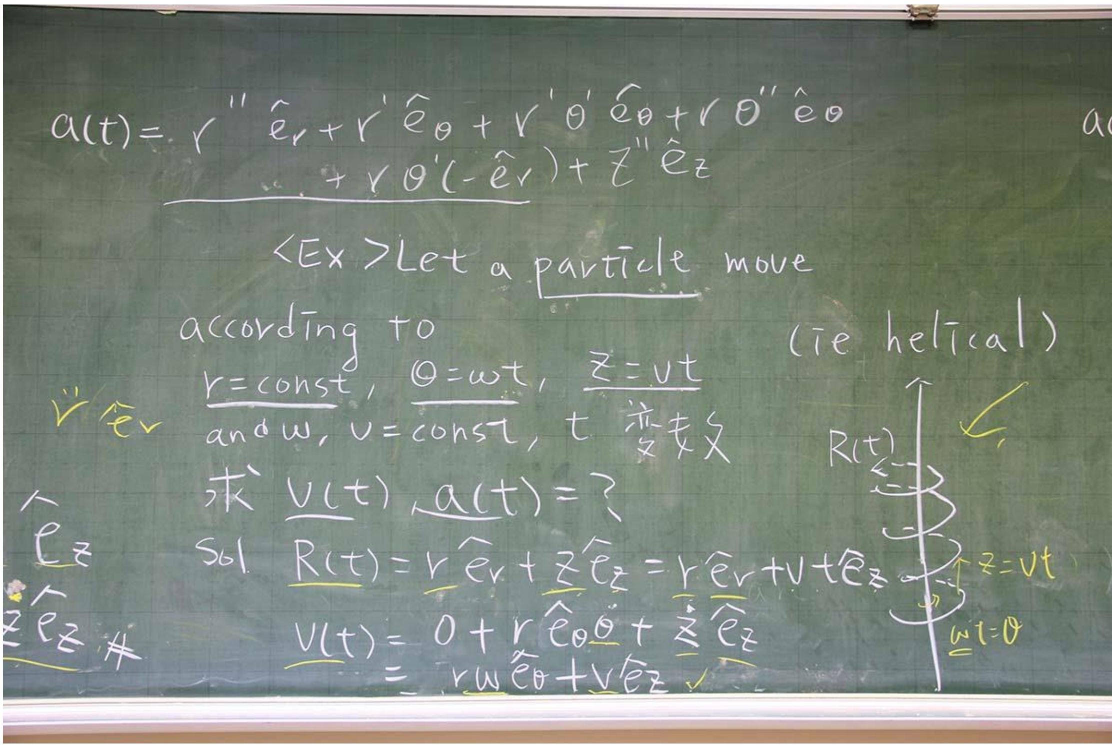
$$\frac{d\hat{e}_\theta}{d\theta} = -\cos\theta\,\hat{i} - \sin\theta\,\hat{j} = -\hat{e}_r \quad \#$$


a(t)= r''
<Ex>Let a particle move
(ie helical)
a(t)=?


3、Cylindrical coordinates (ê_r ê_θ ê_z)
R(t)=r ê_r + z ê_z
const vector
V(t) = ṙ ê_r + r ê_θ θ̇ + ż ê_z
a(t)= r'' ê_r + r' ê_θ + r'θ' ê_θ + rθ'' ê_θ + rθ'(-ê_r) + z'' ê_z
<Ex>Let a particle move
according to
(ie helical)
r=const, θ=ωt, z=vt
and ω, v=const, t 变数
求 V(t), a(t)=?
Sol R(t)= r ê_r + z ê_z = r ê_r + vt ê_z
V(t)= 0 + r ê_θ θ̇ + ż ê_z
= rω ê_θ + v ê_z



3. Cylindrical coordinates ($\hat{e}_r$ $\hat{e}_\theta$ $\hat{e}_z$)
$a(t) =$
$\vec{R}(r, \theta, z)$
z
y
x
$\vec{R}(t) = r\hat{e}_r + z\hat{e}_z$
r, θ, z
$\hat{e}_z$ [const vector]
$\hat{e}_r(\theta)$
$\hat{e}_\theta(\theta)$
$V(t) = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r + \dot{z}\hat{e}_z$
$= \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + \dot{z}\hat{e}_z$ #


$a(t) = r'' \hat{e}_r + r' \hat{e}_\theta + r' \theta' \hat{e}_\theta + r \theta'' \hat{e}_\theta$
$+ r\theta'(-\hat{e}_r) + z'' \hat{e}_z$
<Ex>Let a particle move
according to
(ie helical)
$r = const$, $\theta = \omega t$, $z = vt$
and ω, $v = const$, t 变数
求 $v(t)$, $a(t) = ?$
Sol $R(t) = r\hat{e}_r + z\hat{e}_z = r\hat{e}_r + vt\hat{e}_z$
$v(t) = 0 + r\hat{e}_\theta\dot{\theta} + \dot{z}\hat{e}_z$
$= r\omega\hat{e}_\theta + v\hat{e}_z$
$R(t)$
$z = vt$
$\omega t = \theta$
$r'\hat{e}_r$
$\hat{e}_z$
$z\hat{e}_z$